How to Do Word Problems

Ratio & Proportion Applications

Ratio is a comparison between two numerical values usually written as a fraction in simplest form.

The ratio of A to B is $\frac{A}{B}$, then reduced to the simplest form.

The ratio of A to B can also be written as $A : B$.

Unit Ratio is a ratio in which the denominator is 1.

Example:

Find the ratio of 1.2 to 15.

Solution:

The ratio of 1.2 to 15 is $\frac{1.2}{15}$ which needs to be in a reduced fraction.

$$\frac{1.2}{15} = \frac{1.2(10)}{15(10)} = \frac{12}{150}$$

$$= \frac{6 \cdot 2}{6 \cdot 25} = \frac{\cancel{6} \cdot 2}{\cancel{6} \cdot 25} = \frac{2}{25}$$

So the ratio of 1.2 to 15 can be written as

$$\boxed{\frac{2}{25} \text{ or } 2 : 25}$$

Example:

Find the ratio of $3\frac{1}{5}$ to $1\frac{1}{3}$.

Solution:

The ratio of $3\frac{1}{5}$ to $1\frac{1}{3}$ is $\dfrac{3\frac{1}{5}}{1\frac{1}{3}}$ which needs to be in a reduced fraction.

$$\frac{3\frac{1}{5}}{1\frac{1}{3}} = \frac{\frac{16}{5}}{\frac{4}{3}} = \frac{16}{5} \div \frac{4}{3}$$

$$= \frac{16}{5} \times \frac{3}{4} = \frac{16 \cdot 3}{5 \cdot 4} = \frac{4 \cdot 4 \cdot 3}{5 \cdot 4} = \frac{\cancel{4} \cdot 4 \cdot 3}{5 \cdot \cancel{4}} = \frac{12}{5}$$

Solution(continued):

So the ratio of $3\frac{1}{5}$ to $1\frac{1}{3}$ can be written as.

$$\frac{12}{5} \text{ or } 12 : 5$$

When expressing answers to the ratio problems, do not use mixed numbers, and use decimal numbers only when working with unit ratio.

Example:

A 16-ounce can of corn is priced for \$1.36. Find its unit ratio, that is its price per ounce.

Solution:

The unit ratio can be computed by dividing total cost by total weight.

$$\frac{\$1.36}{16 \text{ ounces}} = \$.085 \text{ per ounce} = 8.5¢ \text{ per ounce}$$

An equation stating that two ratios are equal is called a **proportion**.

In the **proportion equation** $\frac{a}{b} = \frac{c}{d}$, the numbers a and d are called extremes while the numbers b and c are called means.

Cross-Multiplication

In any proportion equation,
the product of the extremes equals the product of the means.

Example:

Solve: $\frac{1.5}{12} = \frac{x}{40}$

Solution:

Using cross-multiplication.

$$\begin{aligned} \frac{1.5}{12} &= \frac{x}{40} \\ 12x &= 40(1.5) \\ x &= \frac{40(1.5)}{12} \\ x &= 5 \end{aligned}$$

$x = 5$ or $\{5\}$

Example:

Solve: $\dfrac{x-4}{x} = \dfrac{3}{5}$

Solution:

Using cross-multiplication.

$$\begin{aligned} \frac{x-4}{x} &= \frac{3}{5} \\ 5(x-4) &= 3(x) \\ 5x-20 &= 3x \\ 5x-3x &= 20 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

$x = 10$ or $\{10\}$

Example:

Solve: $\dfrac{x+3}{2x-1} = \dfrac{1}{2}$

Solution:

Using cross-multiplication.

$$\begin{aligned} \frac{x+3}{2x-1} &= \frac{1}{2} \\ 2(x+3) &= 1(2x-1) \\ 2x+6 &= 2x-1 \\ 2x-2x &= -1-6 \\ 0 &= -7 \end{aligned}$$

Since the last statement is false, there is no solution to this proportion.

Example:

A recipe calls for 2.5 cups of sugar to bake 10 muffins. How many cups of sugar do we need if we like to bake 28 muffins?

Solution:

We first set up the ratio of cups of sugar to the number of muffins, that is $\frac{2.5 \text{ cups of sugar}}{10 \text{ muffins}}$ and $\frac{x \text{ cups of sugar}}{28 \text{ muffins}}$.

Now we equate these two ratios to get the proportion

$$\frac{2.5 \text{ cups of sugar}}{10 \text{ muffins}} = \frac{x \text{ cups of sugar}}{28 \text{ muffins}}$$

Now we use cross-multiplication to solve $\frac{2.5}{10} = \frac{x}{28}$

Solution(continued):

$$\begin{aligned} \frac{2.5}{10} &= \frac{x}{28} \\ 10x &= 2.5(28) \\ 10x &= 70 \\ x &= 7 \end{aligned}$$

So we need 7 cups of sugar to bake 28 muffins.

Example:

A tall building has a shadow of 32.5 feet while at the same time a lightpost with the height 8 feet has a shadow of 12 feet. Use proportion to determine the height of the tree.

Solution:

We first set up the ratio of the height of the object to the length of its shadow, that is

$$\frac{x \text{ feet tall}}{32.5 \text{ feet shadow}} \text{ for the tree and}$$

$$\frac{8 \text{ feet tall}}{12 \text{ feet shadow}} \text{ for the lightpost.}$$

Now we equate these two ratios to get the proportion

$$\frac{x \text{ feet tall}}{32.5 \text{ feet shadow}} = \frac{8 \text{ feet tall}}{12 \text{ feet shadow}}$$

Solution(continued):

Using cross-multiplication to solve

$$\begin{aligned} \frac{x}{32.5} &= \frac{8}{12} \\ 12x &= 32.5(8) \\ 12x &= 260 \\ x &= \frac{260}{12} \\ x &= 21.\overline{6} \end{aligned}$$

The tree is about 21.7 feet tall.